

QCD sum rule determination of the axial-vector coupling of the nucleon at finite temperature

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Abstract

A thermal QCD Finite Energy Sum Rule (FESR) is used to obtain the temperature dependence of the axial-vector coupling of the nucleon, $g_A(T)$. We find that $g_A(T)$ is essentially independent of T , in the very wide range $0 \leq T \leq 0.9 T_c$, where T_c is the critical temperature. While g_A at $T = 0$ is q^2 -independent, it develops a q^2 dependence at finite temperature. We then obtain the mean square radius associated with g_A and find that it diverges at $T = T_c$, thus signalling quark deconfinement. As a byproduct, we study the temperature dependence of the Goldberger-Treiman relation.

The possibility of creating a quark-gluon plasma in relativistic heavy ion collisions has sparked much interest in theoretical predictions for the onset of this state [1]. In addition to the search for unambiguous processes signalling the formation of such a plasma, it is also important to understand the temperature behaviour of hadronic Green's functions and their associated parameters, viz. masses, widths, couplings, etc. The general consensus is that hadronic widths depend strongly on the temperature; in fact they are expected to diverge at some critical temperature T_c , thus signalling quark-gluon deconfinement [2] (hadronic widths are to be understood, in this context, as absorption coefficients determined by the imaginary parts of two-point functions). Thermal three-point functions also provide independent evidence for this phase transition, as the mean square radii happen to increase with increasing temperature, becoming infinite at $T = T_c$ [3].

A recent investigation of the thermal behaviour of the pion-nucleon coupling, in the framework of both the linear sigma model and QCD sum rules [4], showed that as the temperature approaches T_c , $g_{\pi NN}(T)$ vanishes, while the associated radius diverges. Both $g_{\pi NN}(T)$ and $\langle r_{\pi NN}^2 \rangle(T)$ may thus be interpreted as signals for the deconfinement phase transition. In this work we shall determine the temperature behaviour of the axial-vector coupling constant of the nucleon $g_A \equiv g_A(q^2 = 0)$, and the associated radius, using the method of thermal QCD sum rules [5]; specifically, the leading dimension Finite Energy Sum Rule (FESR). However, we shall first discuss our own determination of g_A at $T = 0$, as previous QCD sum rule determinations, dating back many years [6], were the subject of some controversy. We find it possible to reproduce the experimental value of g_A at $T = 0$, which then serves to normalize the finite temperature results. Finally, as a byproduct, we shall use this result to determine the behaviour at finite temperature of the $SU(2)_L \times SU(2)_R$ Goldberger-Treiman relation

$$\frac{f_\pi(T)g_{\pi NN}(T)}{M_N(T)g_A(T)} = 1 + \Delta_\pi(T) \quad . \quad (1)$$

In this relation, $f_\pi(T)$ is known up to $T = T_c$ [7], where it vanishes, $g_{\pi NN}(T)$ behaves qualitatively similarly [4], and $M_N(T)$ is essentially constant up to $T = T_c$ [8]-[9]. The question is then, how big are the thermal corrections to this relation, $\Delta_\pi(T)$ (normalized to $\Delta_\pi(0) = 0$). An equally important chiral-symmetry relation, the Gell-Mann, Oakes and Renner relation (GMOR), has recently been investigated in the framework of thermal chiral perturbation theory [10] and QCD sum rules [11]. There is excellent numerical agreement

between both results, indicating that temperature corrections to the GMOR relation are rather small. It should be kept in mind that a comparison between thermal QCD sum rules results and those from effective theories at finite temperature, e.g. sigma model, chiral perturbation theory, etc., must necessarily be done numerically. The fields involved in the former technique are those of the quarks and gluons, while those of the latter framework are purely hadronic. As a result, expansions in powers of the temperature do not necessarily need to match order by order because the coefficients in these expansions will involve different types of parameters. However, numerical results from both techniques should agree, at least within the range of validity of the low temperature expansion in effective theories (QCD sum rules are in principle valid across the whole range of temperatures). This is precisely what happens with the two analyses of the GMOR relation mentioned above.

We begin by considering the three-point function

$$\Pi_\mu(p, p', q) = i^2 \iint d^4x d^4y \langle 0 | T (\eta_p(x) A_\mu(y) \bar{\eta}_n(0)) | 0 \rangle e^{i(p'x - qy)} , \quad (2)$$

where the charged axial vector current is given by: $A_\mu(x) = \bar{u}(x) \gamma_\mu \gamma_5 d(x)$, while the interpolating currents of the proton and neutron are chosen as [12]

$$\begin{aligned} \eta_p(x) &= \epsilon_{abc} [u^a(x) C \gamma_\mu u^b(x)] \gamma^\mu \gamma_5 d^c , \\ \eta_n(x) &= -\epsilon_{abc} [d^a(x) C \gamma_\mu d^b(x)] \gamma^\mu \gamma_5 u^c . \end{aligned} \quad (3)$$

The axial-vector coupling of the nucleon, $g_A(q^2)$, is defined through

$$\langle N(p_2) | A_\mu(0) | N(p_1) \rangle = \bar{u}(p_2) [\gamma_\mu \gamma_5 g_A(q^2) + q_\mu \gamma_5 h_A(q^2)] u(p_1) , \quad (4)$$

with $q_\mu = (p_2 - p_1)_\mu$. The coupling of the interpolating currents, Eq.(3), to the nucleon is

$$\langle 0 | \eta(0) | N(p) \rangle = \lambda_N u(p) . \quad (5)$$

Inserting a complete set of intermediate nucleon states into Eq.(2), one obtains the hadronic representation

$$\Pi_\mu(p, p', q) = \frac{\lambda_N^2}{(p^2 - M_N^2)(p'^2 - M_N^2)} (\not{p}' + M_N) T_\mu (\not{p} + M_N) , \quad (6)$$

where

$$T_\mu = [\gamma_\mu \gamma_5 g_A(q^2) + q_\mu \gamma_5 h_A(q^2)] , \quad (7)$$

and the following expansion holds

$$\begin{aligned}
(\not{p}' + M_N)T_\mu(\not{p} + M_N) &= g_A(q^2) [-2 \not{p}' p_\mu \gamma_5 + \not{p}' \not{p} \gamma_\mu \gamma_5 + (\not{p}' + \not{p}) \gamma_\mu \gamma_5 M_N \\
&\quad - 2p_\mu \gamma_5 M_N + M_N^2 \gamma_\mu \gamma_5] \\
&\quad + h_A(q^2) [-\not{p}' \not{p} + \not{q} M_N + M_N^2] q_\mu \gamma_5. \tag{8}
\end{aligned}$$

Since we are only interested in g_A , we need to extract tensor structures which are not multiplied by h_A ; a suitable candidate being the structure

$$(\not{p}' + \not{p}) \gamma_\mu \gamma_5. \tag{9}$$

The relevant term of the imaginary part of the (hadronic) correlator is then

$$\begin{aligned}
Im \Pi_\mu(s, s', q^2)|_{\text{HAD}} &= -\lambda_N^2 g_A(q^2) M_N \pi^2 \delta(s - M_N^2) \delta(s' - M_N^2) (\not{p}' + \not{p}) \gamma_\mu \gamma_5 \\
&\quad + \Theta(s - s_0) \Theta(s' - s'_0) Im \Pi_\mu(s, s', q^2)|_{\text{QCD}}, \tag{10}
\end{aligned}$$

where $s = p^2$, $s' = p'^2$, and we have added the hadronic continuum, modelled by perturbative QCD, starting at thresholds $s = s_0$ and $s' = s'_0$. Considering the contribution to the correlator from perturbative QCD, we obtain

$$\begin{aligned}
\Pi_\mu(p, p', q)|_{\text{PQCD}} &= -24i^2 \int \frac{d^4 k_1 d^4 k_2}{(2\pi)^8} \gamma_\alpha S_F(k_2) \gamma_\beta S_F(k_1 - q) \gamma_\mu \\
&\quad \times S_F(k_1) \gamma^\alpha S_F(p' - k_1 - k_2) \gamma^\beta \gamma_5. \tag{11}
\end{aligned}$$

Taking the imaginary part of this expression, and evaluating the integrals, it turns out that there are no terms proportional to the tensor structure of Eq.(9). Turning to the non-perturbative part, we find the quark condensate contribution to the correlator to be

$$\begin{aligned}
\Pi_\mu(p, p', q)|_{\text{QCD}} &= 2i^3 \langle \bar{d}d \rangle \left[\int \frac{d^4 k}{(2\pi)^4} \gamma_\alpha S_F(k) \gamma_\beta \gamma_\mu S_F(q) \gamma^\alpha S_F(p' - k - q) \gamma^\beta \gamma_5 \right. \\
&\quad \left. - \int \frac{d^4 k}{(2\pi)^4} \gamma_\alpha \gamma_\beta S_F(k - q) \gamma_\mu S_F(k) \gamma^\alpha S_F(p' - k) \gamma^\beta \gamma_5 \right] \\
&\quad + \langle \bar{u}u \rangle \left[\int \frac{d^4 k}{(2\pi)^4} \gamma_\alpha S_F(p' - k) \gamma_\beta S_F(k - q) \gamma_\mu S_F(k) \gamma^\alpha \gamma^\beta \gamma_g \right. \\
&\quad \left. + \int \frac{d^4 k}{(2\pi)^4} \gamma_\alpha S_F(k) \gamma_\beta S_F(q) \gamma_\mu \gamma^\alpha S_F(p' - k) \gamma^\beta \gamma_5 \right] \tag{12}
\end{aligned}$$

Taking the imaginary part, and keeping only terms proportional to the relevant tensor structure Eq.(9), and which are non-vanishing in the limit $q^2 \rightarrow 0$, we obtain, after assuming $\langle \bar{u}u \rangle \simeq \langle \bar{d}d \rangle \equiv \langle \bar{q}q \rangle$,

$$Im \Pi_\mu(p, p', q)|_{\text{QCD}} = \frac{1}{12\pi} \langle \bar{q}q \rangle (\not{p} + \not{p}') \gamma_\mu \gamma_5. \quad (13)$$

Next, using Cauchy's theorem, and assuming quark-hadron duality, the lowest dimensional FESR for g_A reads

$$\int_0^{s_0} \int_0^{s'_0} ds \, ds' \, Im \Pi_\mu(s, s')_{\text{HAD}} = \int_0^{s_0} \int_0^{s'_0} ds \, ds' \, Im \Pi_\mu(s, s')_{\text{QCD}}. \quad (14)$$

From this FESR one then obtains the relation

$$g_A = -\frac{s_0 s'_0}{12\pi^3} \frac{\langle \bar{q}q \rangle}{\lambda_N^2 M_N}. \quad (15)$$

At first sight, this result hardly looks like a prediction for g_A , since s_0 , s'_0 , and λ_N are a-priori unknown. However, since the double dispersion in $p^2 = s$ and $p'^2 = s'$, used in obtaining Eq.(15), refers to the nucleonic legs of the three-point function, it is reasonable to set $s_0 = s'_0$. At the same time, a QCD FESR analysis of the two-point function involving nucleonic currents [8] yields the following relations

$$\lambda_N^2 = \frac{s_0^3}{192\pi^4}, \quad \lambda_N^2 M_N = -\frac{\langle \bar{q}q \rangle}{8\pi^2} s_0^2, \quad (16)$$

where, in principle, the numerical value of the asymptotic freedom threshold s_0 does not have to be the same as that in Eq.(15). In fact, if one were to assume them to be equal, then Eqs.(13) and (14) would imply $g_A = 8/12\pi$, a value far too small. Without any attempt at extracting a *precision* value of g_A , it is rewarding, though, to find that the experimental value $g_A = 1.26$ can be reproduced in this framework if $\lambda_N^2 \simeq 3.1 \times 10^{-4} \text{ GeV}^6$, $s_0 = 3.7 \text{ GeV}^2$, and the standard value $\langle \bar{q}q \rangle = -0.01$, are used in Eq. (15). These values of λ_N^2 and s_0 are close enough to those resulting from the two-point function channel, at least for the purpose of the present work, which is to obtain the temperature dependence of g_A and its mean square radius, rather than a prediction for $g_A(T=0)$.

The finite temperature corrections to g_A are obtained by inserting the thermal Dolan and Jackiw [13] propagators, and allowing for the temperature variation of $\langle \bar{q}q \rangle$, λ_N and s_0 . For $\langle \bar{q}q \rangle_T$ and $\lambda_N(T)$ we shall use the results of [7] and of [8], respectively. The temperature dependence of s_0 was first obtained in [14], and later improved in [15]. It turns out that for a wide range of temperatures not too close to T_c , say $T < 0.8T_c$, the following scaling relation holds to a good approximation

$$\frac{f_\pi^2(T)}{f_\pi^2(0)} \simeq \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} \simeq \frac{s_0(T)}{s_0(0)} . \quad (17)$$

The appropriate contribution to the thermally corrected QCD spectral function becomes

$$Im \Pi_\mu(p, p', q) = \frac{\langle \bar{q}q \rangle}{48\pi} (\not{p} + \not{p}') \gamma_\mu \gamma_5 [f(p, T) + f(p', T)] , \quad (18)$$

where

$$f(p, T) = \int_{-1}^1 dx \left[1 - n_F \left(\frac{|p_0 - |\vec{p}|x|}{2} \right) - n_F \left(\frac{|p_0 + |\vec{p}|x|}{2} \right) \right] , \quad (19)$$

with $n_F(x) = (1 + e^x)^{-1}$, and $f(p', T)$ is similarly defined. Finally, we obtain the sum rule for g_A at finite temperature:

$$g_A(T) = -\frac{\langle \bar{q}q \rangle}{48\pi^3} \frac{1}{\lambda_N^2 M_N} \int_0^{s_0(T)} ds \int_0^{s'_0(T)} ds' [f(p, T) + f(p', T)] . \quad (20)$$

In order to evaluate the integrals one needs to choose a specific frame, for example the (rest) frame $\vec{p} = 0$. In this case, the components of the four vectors p and p' may be expressed in terms of s , s' and q^2 . Other choices of frames give essentially the same results. A numerical evaluation of $g_A(T)$ is presented in Fig. 1. As can be seen from this figure, g_A is basically T-independent, and it clearly does not vanish as the critical temperature is approached. In this sense, g_A does not represent a signal for the deconfinement phase transition. We turn now to the mean square radius $\langle r_A^2 \rangle_T$ associated with g_A , and defined as

$$\langle r_A^2 \rangle_T = 6 \frac{\partial}{\partial q^2} \ln g_A(q^2, T) |_{q^2=0} . \quad (21)$$

This radius is non-zero at finite temperature due to the q^2 -dependence of the arguments of the thermal Fermi factors. After evaluating the logarithmic derivative of Eq.(20) one obtains

$$\begin{aligned} \langle r_A^2 \rangle_T = & \left\{ \int_0^{s_0} \int_0^{s'_0} ds \, ds' [f(p, T) + f(p', T)] \right\}^{-1} \int_0^{s_0(T)} ds \int_0^{s'_0(T)} ds' \int_{-1}^1 dx \\ & \times \frac{6}{2T\sqrt{s}} \left(1 + x \frac{p'_0}{|\vec{p}'|} \right) e^{\frac{p'_0 + |\vec{p}'|x}{2T}} \left[n_F \left(\frac{p'_0 + |\vec{p}'|x}{2} \right) \right]^2. \end{aligned} \quad (22)$$

This is plotted in fig.2, which shows that the radius diverges as the critical temperature is approached. This kind of behaviour has been obtained previously for other radii [3], [4], and it may be interpreted as (analytic) evidence for quark deconfinement.

Finally, we can use our result for g_A at non-zero temperature to evaluate the validity of the GTR, Eq.(1). Results for the mass of the nucleon show that it has very little variation with temperature, and so we shall assume that it is constant [8]-[9]. Using the result of [7] for f_π at finite temperature, together with our previous results for $g_{\pi NN}(T)$ [4], and our current result for $g_A(T)$, we can determine the thermal correction to the GTR, $\Delta_\pi(T)$ defined in Eq.(1). In fig.3 we present a plot of $1 + \Delta_\pi(T)$ against T/T_c , which indicates that the GTR is approximately correct until about $T \simeq 0.9 T_c$, where it breaks down.

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Figure Captions

Figure 1. The coupling $g_A(T)$, from Eq. (20), as a function of T/T_c .

Figure 2. The temperature dependence of the mean square radius, Eq.(22).

Figure 3. Deviation from the Goldberger-Treiman relation, Eq.(1), as a function of T/T_c .

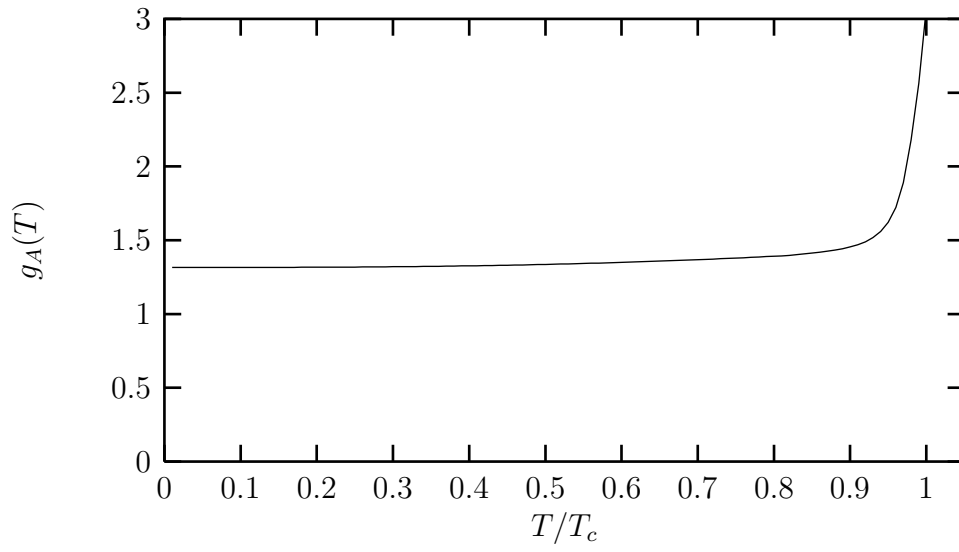


Figure 1:

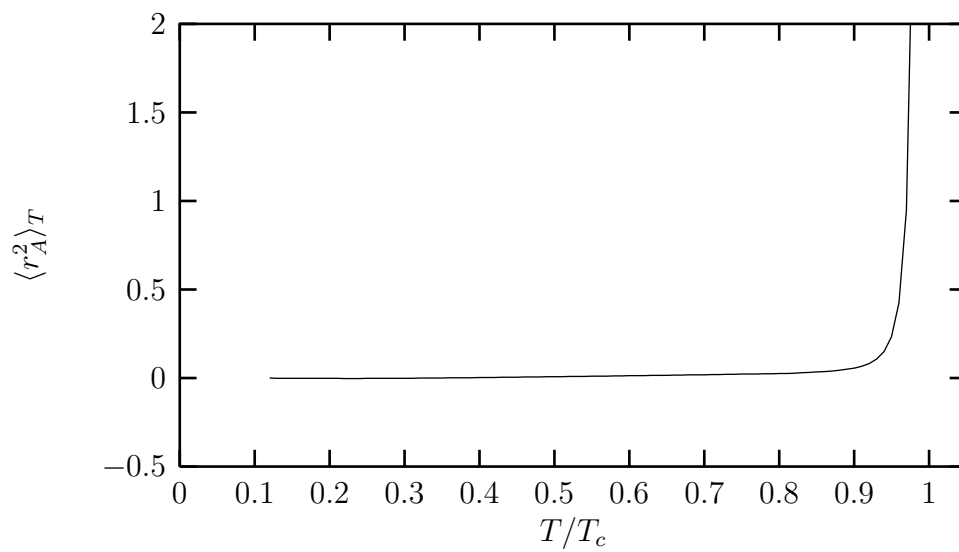


Figure 2:

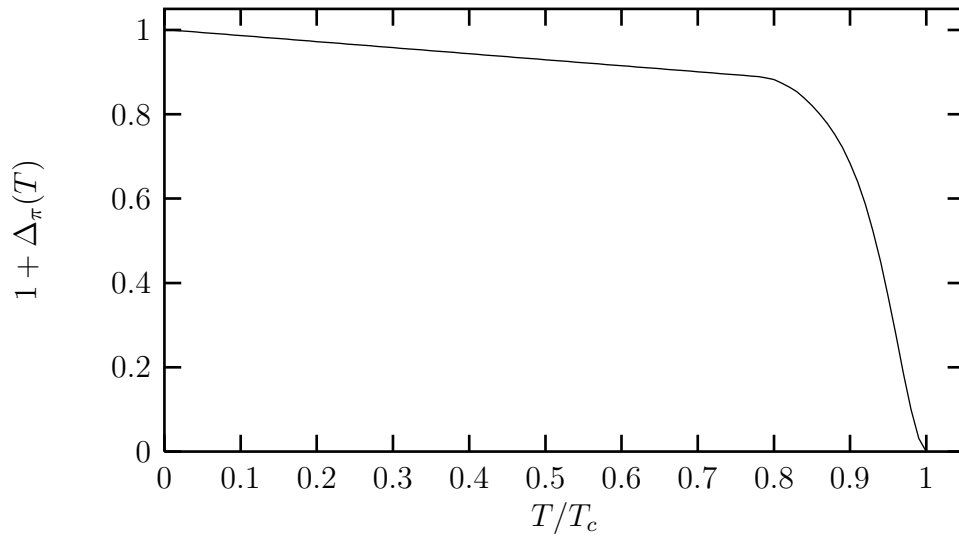


Figure 3: